Randomized Exploration for Reinforcement Learning with General Value Function Approximation

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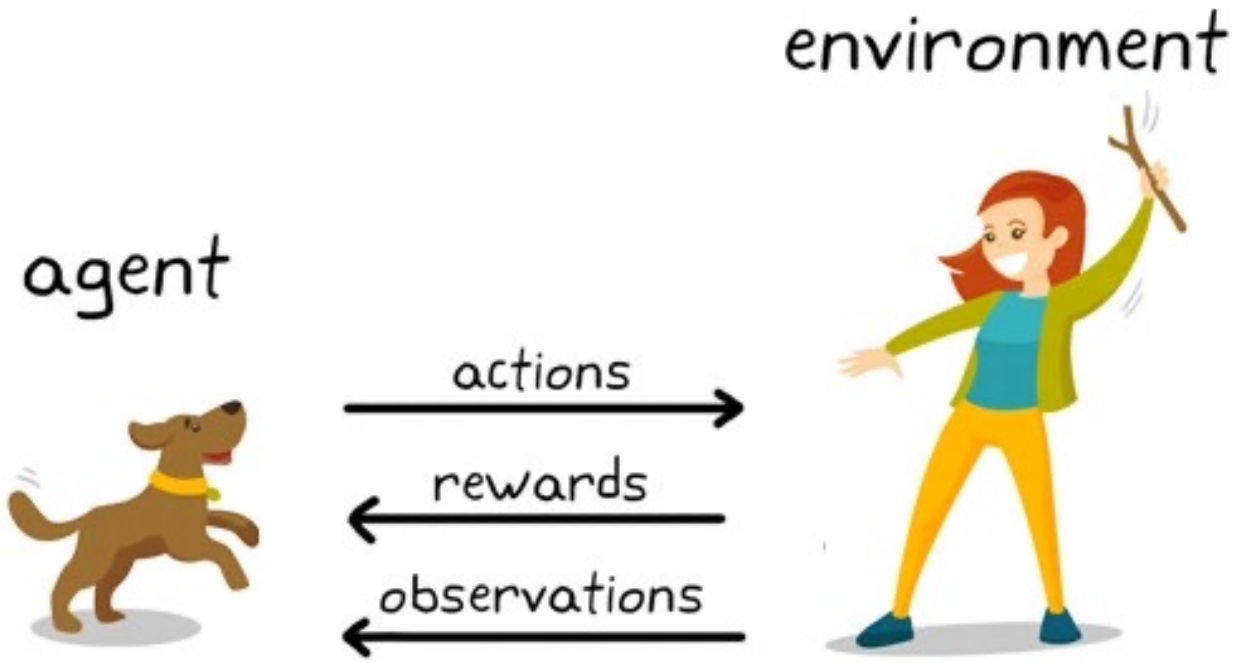
Mila

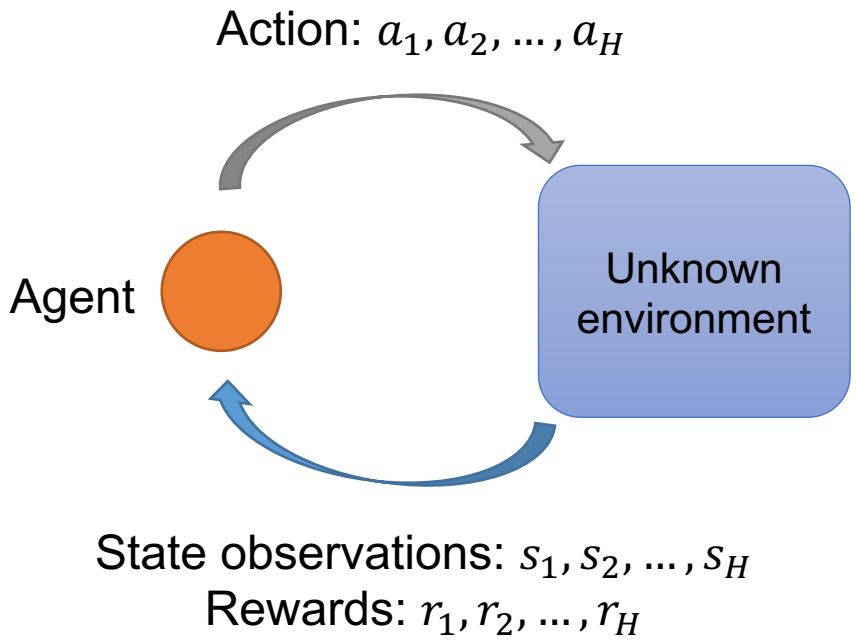
Reinforcement Learning

Learn to interact with an unknown environment through trial and error



Reinforcement Learning

Learn to interact with an unknown environment through trial and error

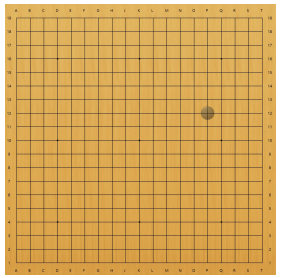


Goal: maximize cumulative reward for a horizon H

Value: $\mathrm{E}[r_1 + r_2 + r_3 + \cdots r_H]$

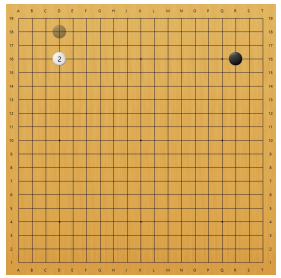
Long term effect needs to be considered.

Reinforcement Learning



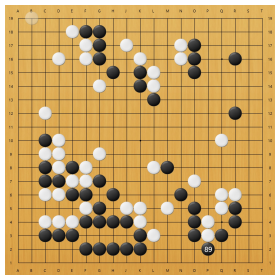
s_0

$a_0 \leftarrow$ move 1
$r_0 \leftarrow 0$



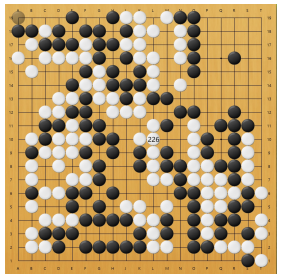
s_1

$a_1, a_2, \ldots$
$r_1, r_2 \ldots \leftarrow 0$



s_{89}

$a_{90}, a_{91}, \ldots$
$r_{90}, r_{91} \ldots \leftarrow 0$



s_{226}

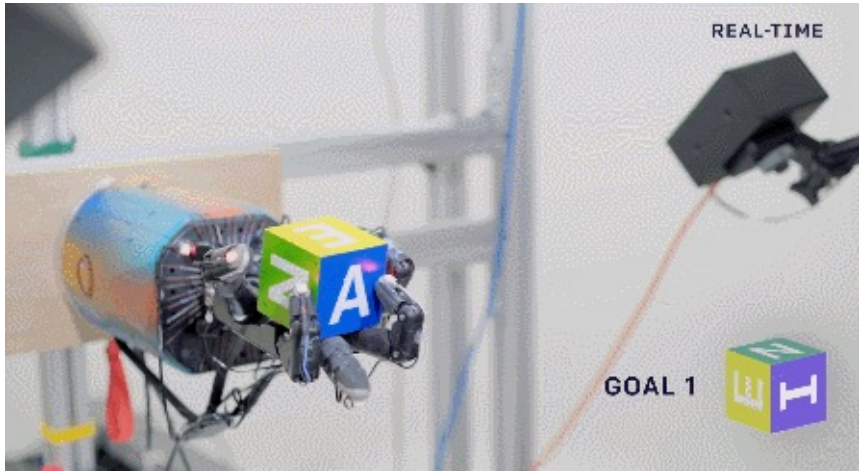
Wining: $r_{226} \leftarrow 1$

Losing: $r_{226} \leftarrow 0$

Actions?

States?

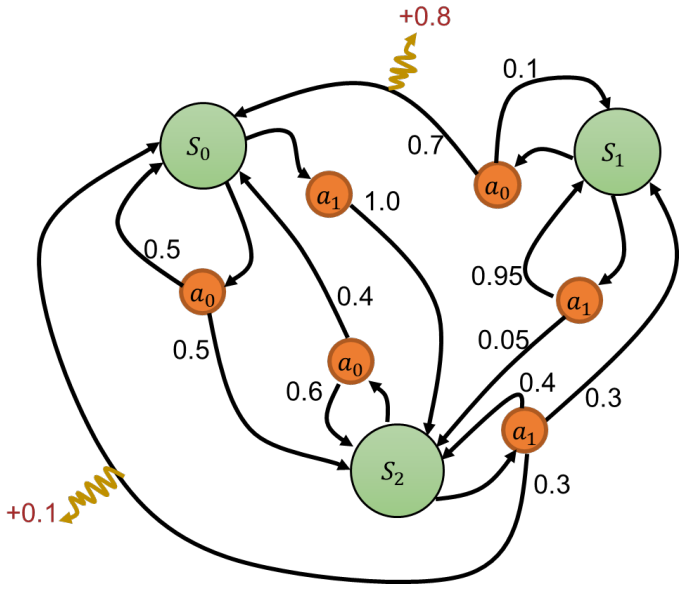
Rewards?



OpenAI Arm

Markov Decision Process (MDP)

- Environment is unknown
  - States: S; actions: A
  - Reward: $r(s, a) \in [0, 1]$
  - **Unknown** state transition: $P_h(\cdot\,|s, a)$
  - Horizon: H (a large number)
  - Goal: optimal policy $\pi^*: S \to \Delta_A$

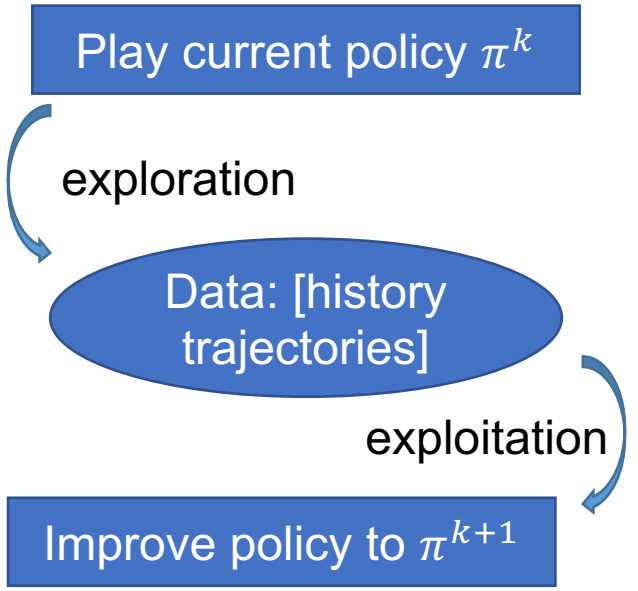


$$\max_{\pi} \mathrm{E}\left[r_1\left(s_1, \pi(s_1)\right) + r_2\left(s_2, \pi(s_2)\right) + \cdots r_H\left(s_H, \pi(s_2)\right)\right] =: Q^{\pi}$$

$$s_i \sim P(\cdot\,|s_{i-1}, \pi(s_{i-1}))$$

Theories of RL on MDP

- Exploration + exploitation [Kearns & Singh 2002, Jaksch et al. 2010]
  - Learn from scratch
  - Exploitation: optimize policy based on existing data
  - Exploration: collect new info about the environment
    - *Regret: average error v.s. optimal policy*
- Focus has been on Tabular RL
  - Does not scale in practical problem
  - Provides sanity check for exploration algorithm
  - In deep RL, the default is ϵ-greedy exploration



Does tabular algorithm work in practice?

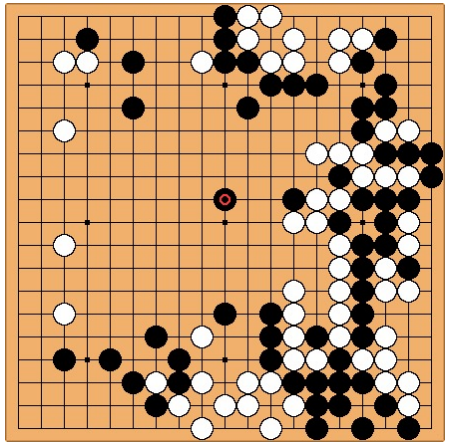
- Number of episodes required to get a good π

$$\widetilde{\Theta}[|S||A|\mathrm{poly}(H)]$$

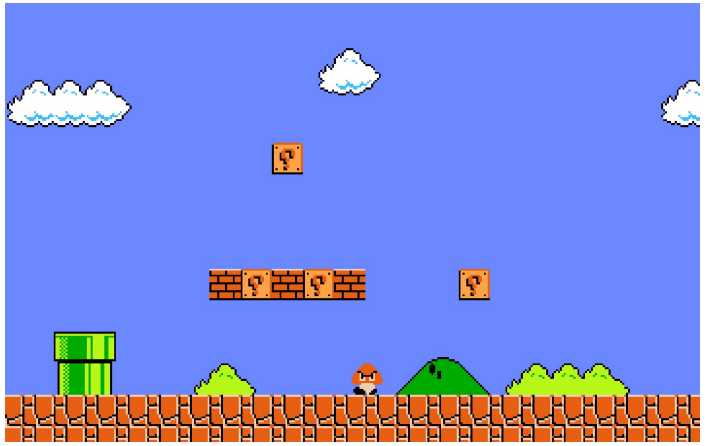
[Jin et al'2018] [Azar et al' 2017][…]

- Curse of Dimensionality

S



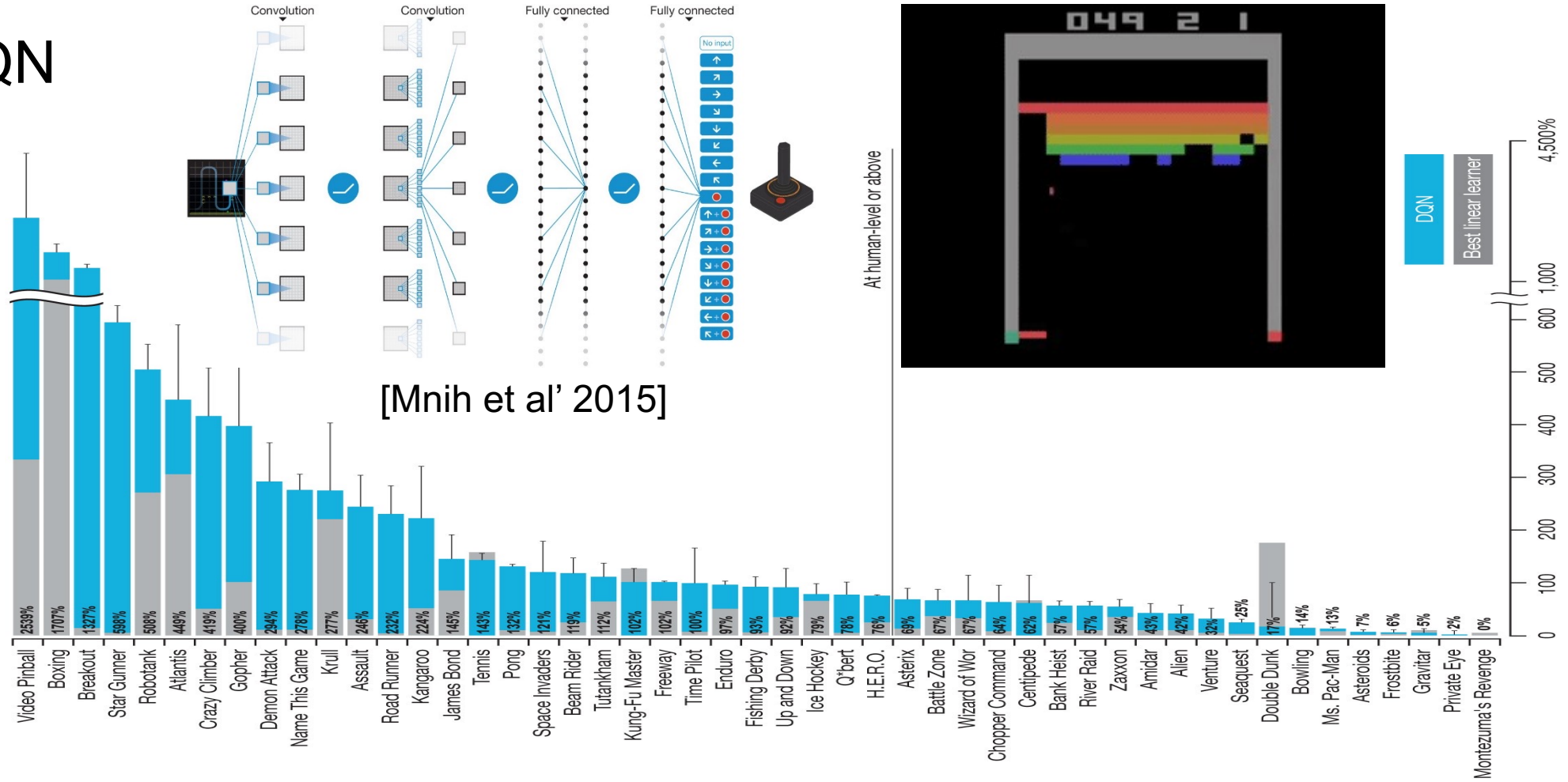
$|S| = 3^{361}$



$|S| \geq 256^{256\times240}$

Function Approximation in Practice

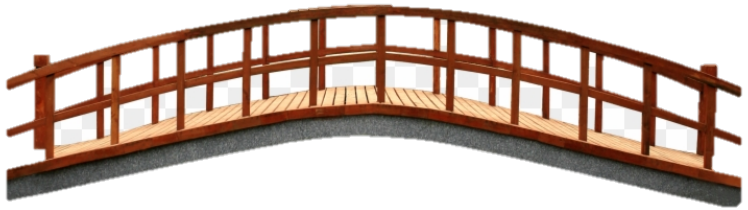
- DQN



[Mnih et al' 2015]

Limitations? **Huge** number of training samples. Hard to **understand**. No **theoretical** guarantee.

RL Theory v.s. Practice



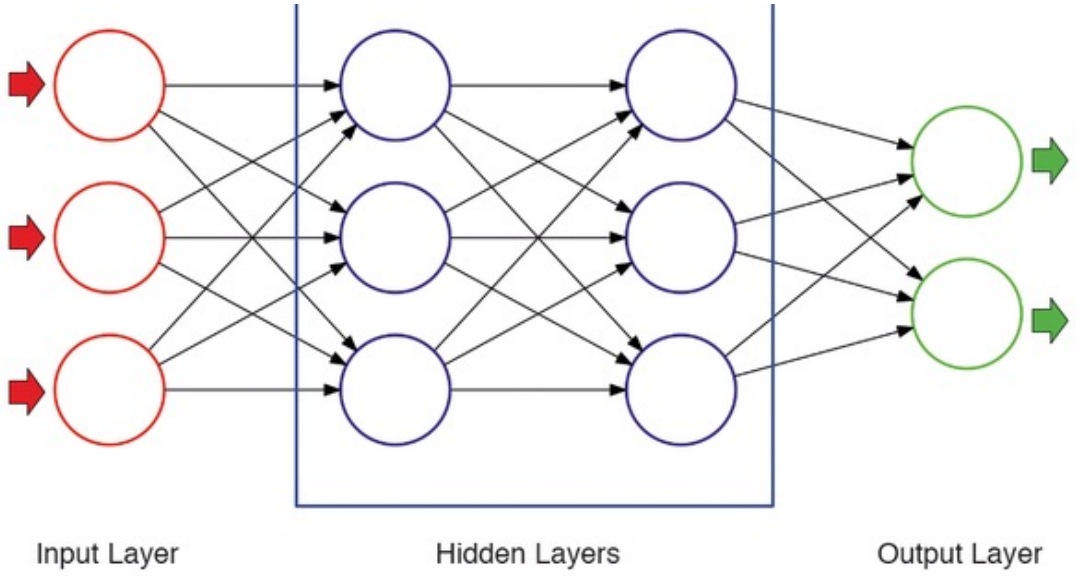
- Theory
  - Markov decision process
    - Finite state space S
    - Finite action space A
    - Finite horizon H
  - Many theoretical results
    - Mostly tabular – well understood
    - Not scalable

- Practice
  - Infinite state space
  - Function approximation via Deep Neural Networks
  - Many empirical results
    - Little understanding
    - No guarantee

Function Approximation

- Find a function class to approximate $Q^*(s,a)$ or π^*

(s,a)



$Q^*(s,a)$

$\pi^*(s,a)$

- Generalization ability
  - Infer values/policies for unseen (s, a)

Linear Function Approximation

- Need correct features
  - Features are given: $\phi(s, a) \to R^d$

ϕ ( , (Action Left)) = (3 question marks, 1 enemies, 4 bushes, 1 chimney, …)

  - Bad features requires **exponential** time/sample to learn

    [Du-Kakade-Wang-Yang' 20] [Van Roy & Dong' 20] [Lattimore et al' 20] [Weisz et al' 20]

- Good features
  - Linear MDP [Yang & Wang' 19]:

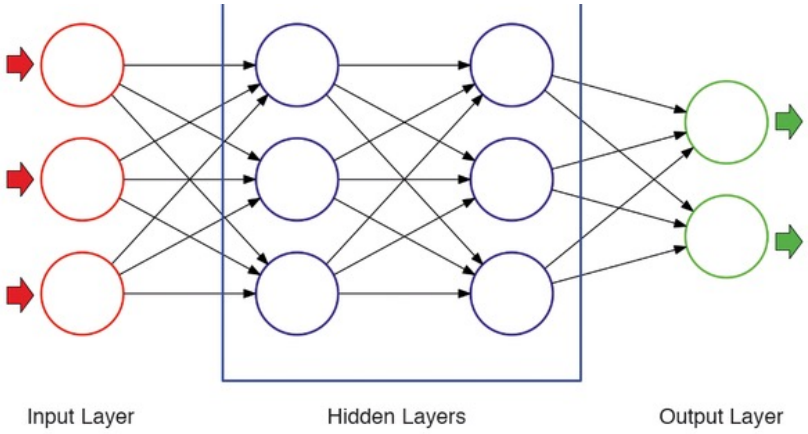
    efficient algorithm: [Jin et al' 20]
  - Low-bellman error [Zanette et al' 20]
  - Low-bellman rank [Jiang et al' 17]

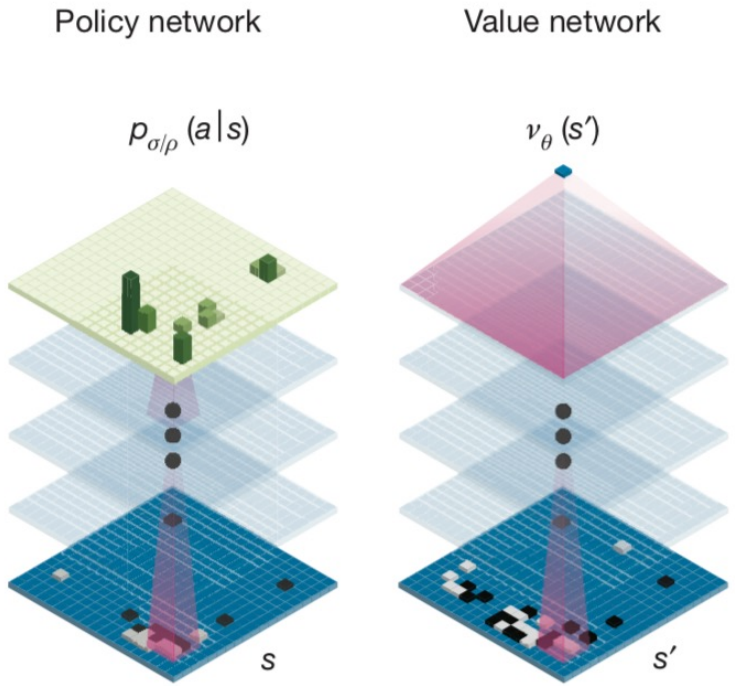
$$P(s'|s,a) = \sum_{k\in[K]} \phi_k(s,a)^\top \psi_k(s')$$

Time efficient

General function approximation

- No features are given
- Function class $\mathcal{F}$
  - Might be parametric
  - $f(s,a)$ may rep. $Q^*(s,a)$
- Used in practice





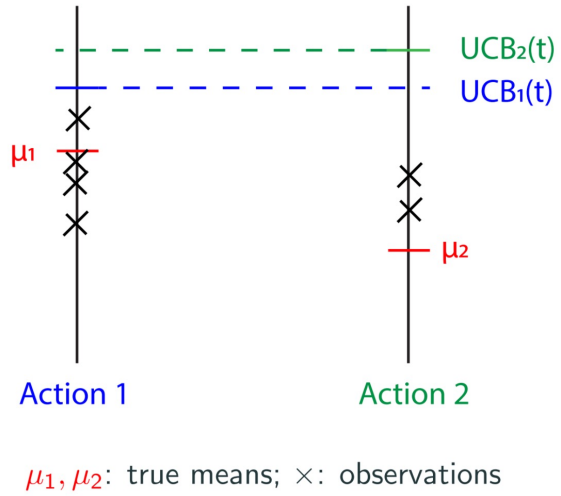
[Silver et al 2016]

Goals for RL:

- Efficient algorithms with practical potentials
- Theoretical guarantees for special cases

Strategies for Exploration

- Optimism in the face of uncertainty:
  - Upper Confidence Bound (UCB)



- Thompson Sampling
  - One of the oldest heuristics for balancing exploration exploitation trade-off. (Thompson, 1933)
  - Randomly select an action according to the probability of it being the optimal action.
  - PSRL = Thompson Sampling for MDPs. (Strens 2000)
  - Sample MDP from posterior, apply policy for an entire episode.

Randomized value functions

- Key idea: generate approximate posterior samples
  - Use standard value learning algorithms (LSVI, DQN, …)
  - Fit to randomly perturbed data

- Theory for tabular representation + LSVI:
  - Worst-case regret bound for Gaussian noise (Russo 2019)

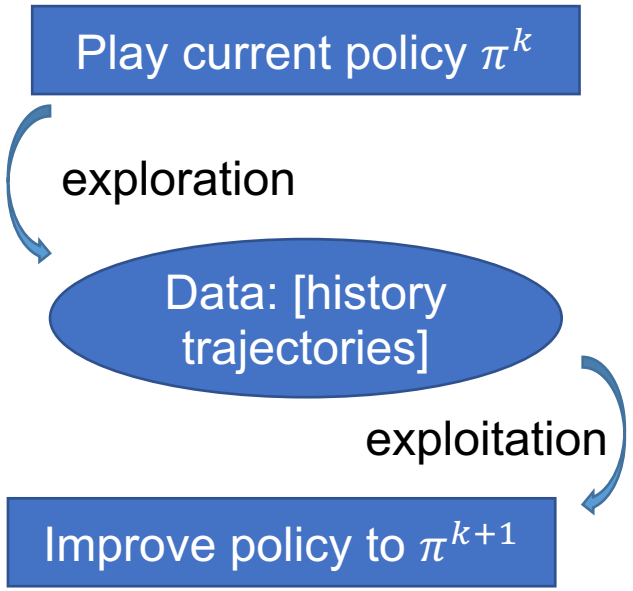
$$Regret(T) \leq \tilde{O}(H\sqrt{S^3AT})$$

- . Computational results with generalization
  - Parameterized representation for Q(s,a)
  - Scalable unlike UCB based methods or posterior sampling
  - Approximate posterior inference is good enough for efficient exploration.

Current limitations

- No theoretical result for RVF with general function approximation
  - Limited to empirical results only (Bootstrapped DQN, Ensemble sampling)
- Lack of unification between OFU and Thompson Sampling
  - Can we combine both principle for algorithm design?
- Bypassing UCB bonus in applying OFU principle
  - UCB bonus is not scalable
  - For GFA, requires complicated sensitivity sampling scheme [Wang et al, 2020]

LSVI for Online RL with **General** VFA



- Initialize an arbitrary $Q^0 \leftarrow 0$
  - For episode $k = 1, 2, \ldots K$:
    - Solve for Q_h^k using LSVI on the history

$$\theta_h^k \leftarrow \text{argmin}_w \sum_t \left[f_w(s_t, a_t) - \left(r(s_t, a_t) + \max_a Q_{h+1}^k(s_{t+1}, a) \right) \right]^2$$

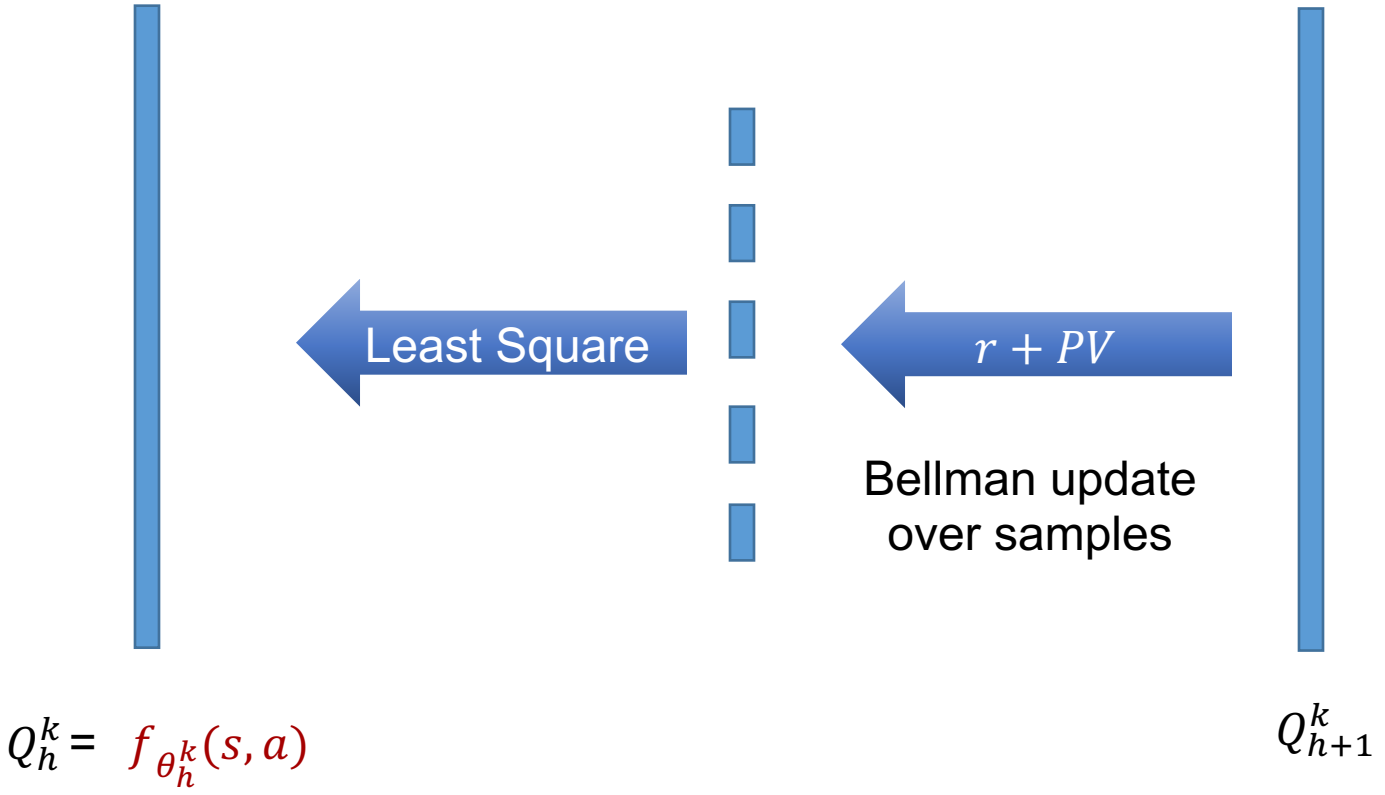
$$Q_h^k(s, a) = f_{\theta_h^k}(s, a)$$

- Collect a trajectory of data $\pi_h^k(s) \leftarrow \text{argmax}_a Q_h^k(s, a)$

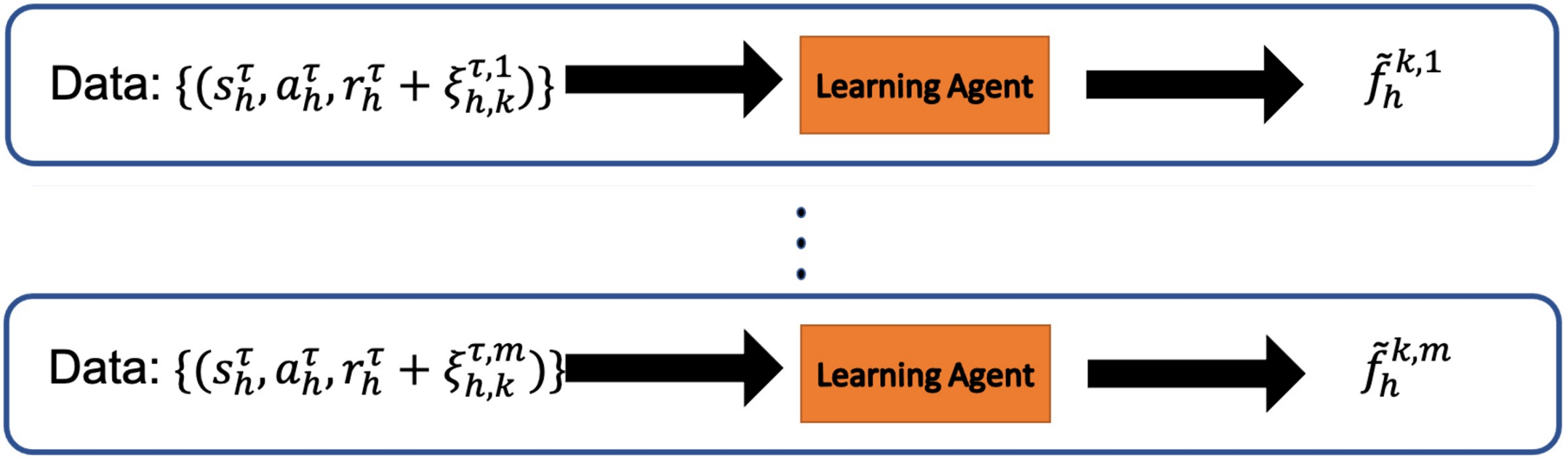
$$(s_1^k, a_1^k, r_1^k) \rightarrow (s_2^k, a_2^k, r_2^k) \rightarrow (s_3^k, a_3^k, r_3^k) \rightarrow \cdots (s_H^k, a_H^k, r_H^k)$$

LSVI as Approximate Dynamic Programming (ADP)

- Each iteration solves



Optimistic Sampling



$$Q_h^{k,m}(\cdot,\cdot) = \tilde{f}_h^{k,m}(\cdot,\cdot)$$

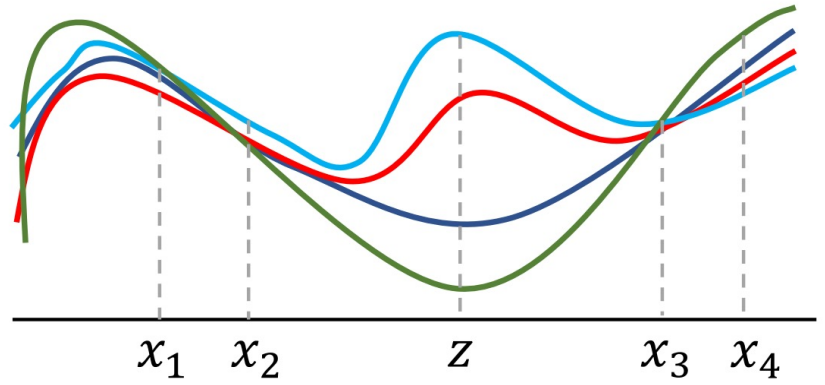
$$Q_h^k(\cdot,\cdot) = \min\{\max_{m\in[M]} Q_h^{k,m}(\cdot,\cdot), \mathrm{H} - \mathrm{h} + 1\}$$

Theory for General functions

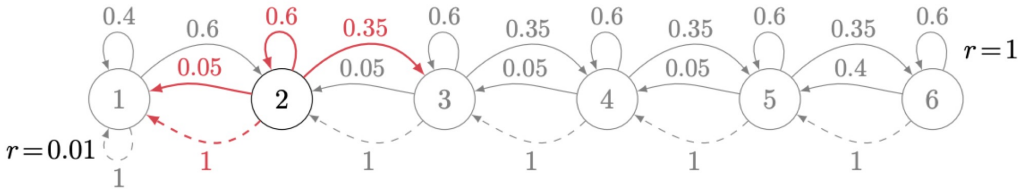
- Assumption: $r + PV \in \mathcal{F}, \forall V$
  - Realizability: The function set is the “image” of Bellman projection
  - Corresponding to linear MDP for linear setting
- Eluder dimension [Russo&Van Roy’ 2013]
  - d_E: the longest determination sequence of the function set
  - d-dim linear / generalized linear: $\approx d$

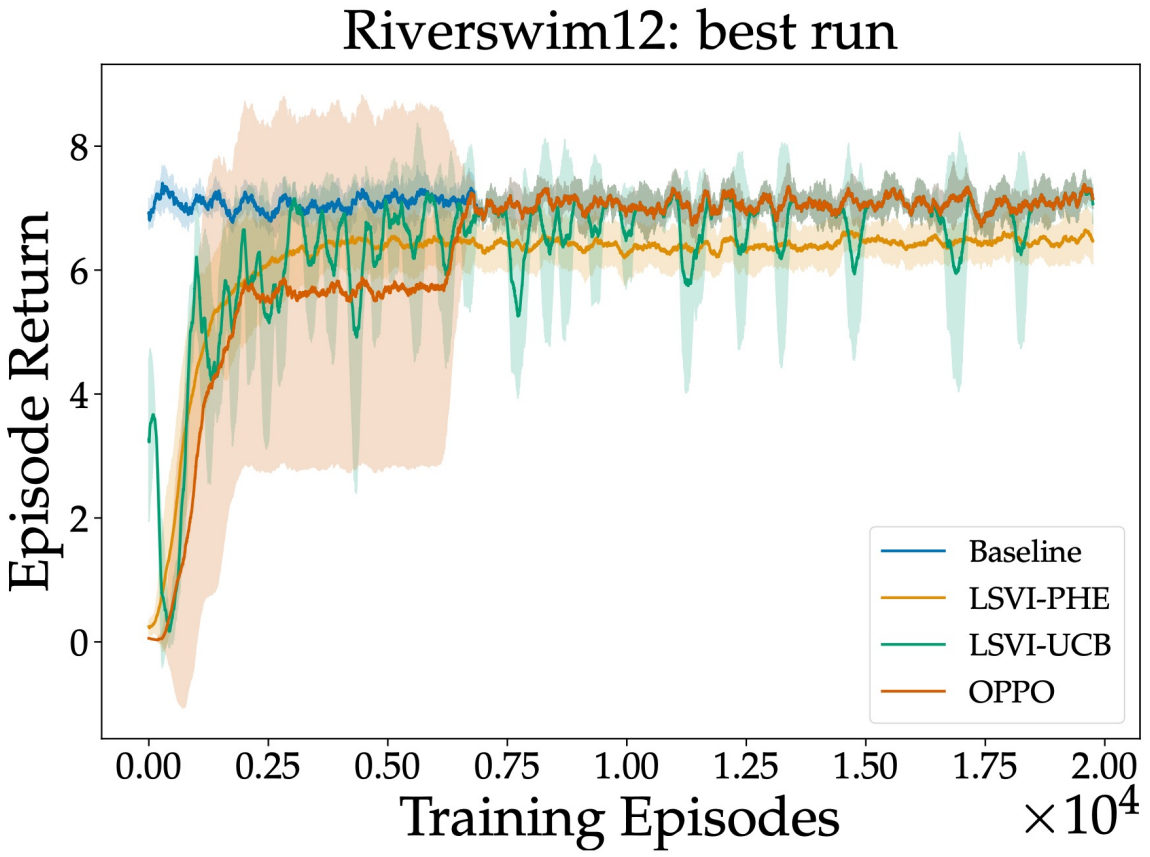
Theorem:

LSVI-PHE with **optimistic sampling** satisfies regret bound of $\boldsymbol{O}\left(\mathrm{poly}(\boldsymbol{d_E H})\sqrt{\boldsymbol{T}}\right)$ with high probability

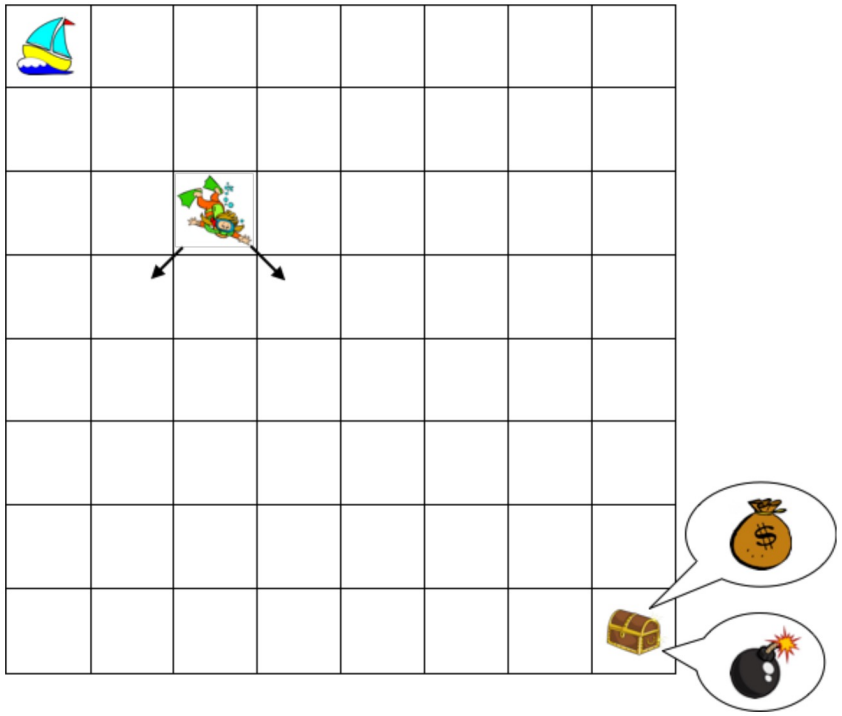


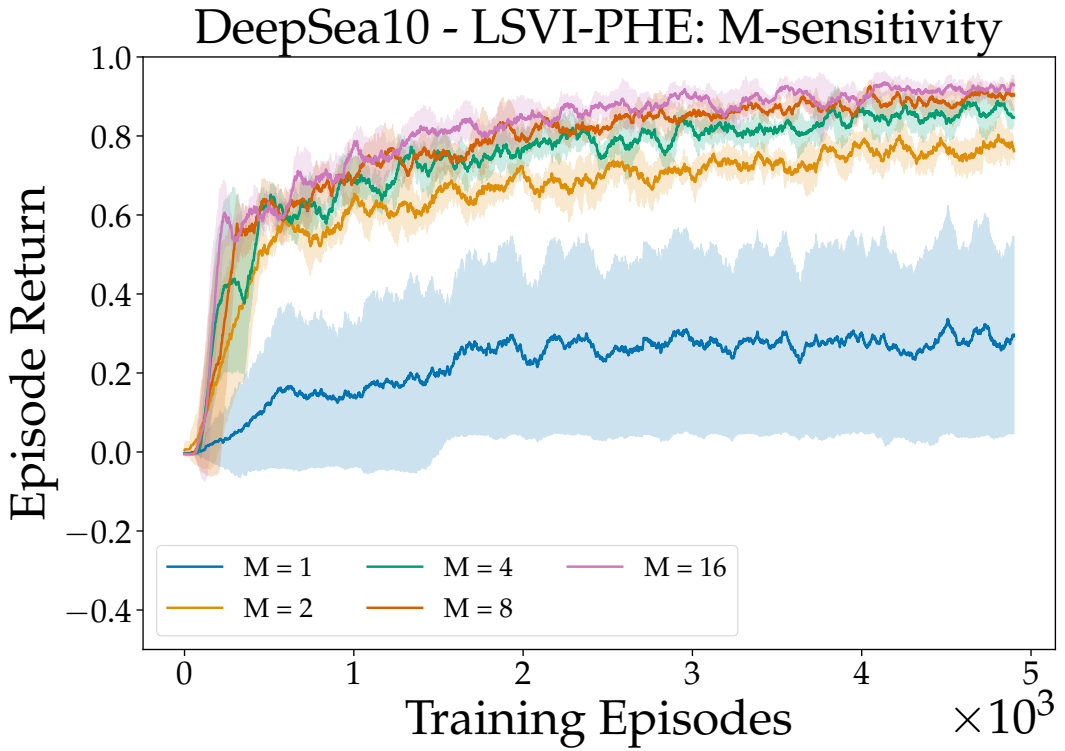
Riverswim:


0.4
0.6
0.6
0.35
0.6
0.35
0.6
0.35
0.6
0.35
0.6
0.05
0.05
0.05
0.05
0.4
$r=1$
1
2
3
4
5
6
$r=0.01$
1
1
1
1
1
1

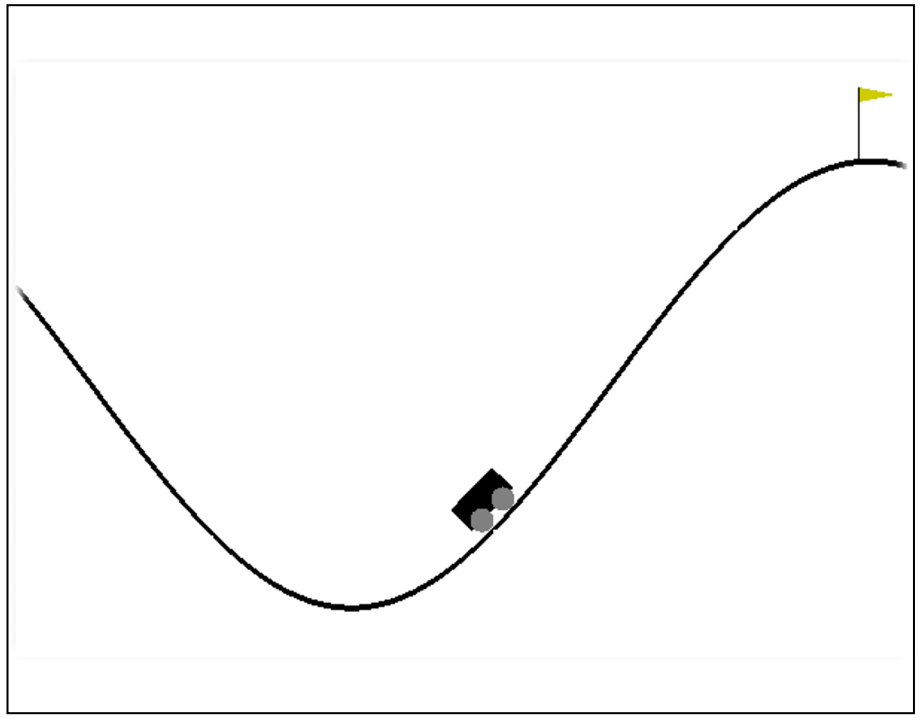

Riverswim12: best run
Episode Return
Training Episodes
$\times 10^4$
0
2
4
6
8
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Baseline
LSVI-PHE
LSVI-UCB
OPPO

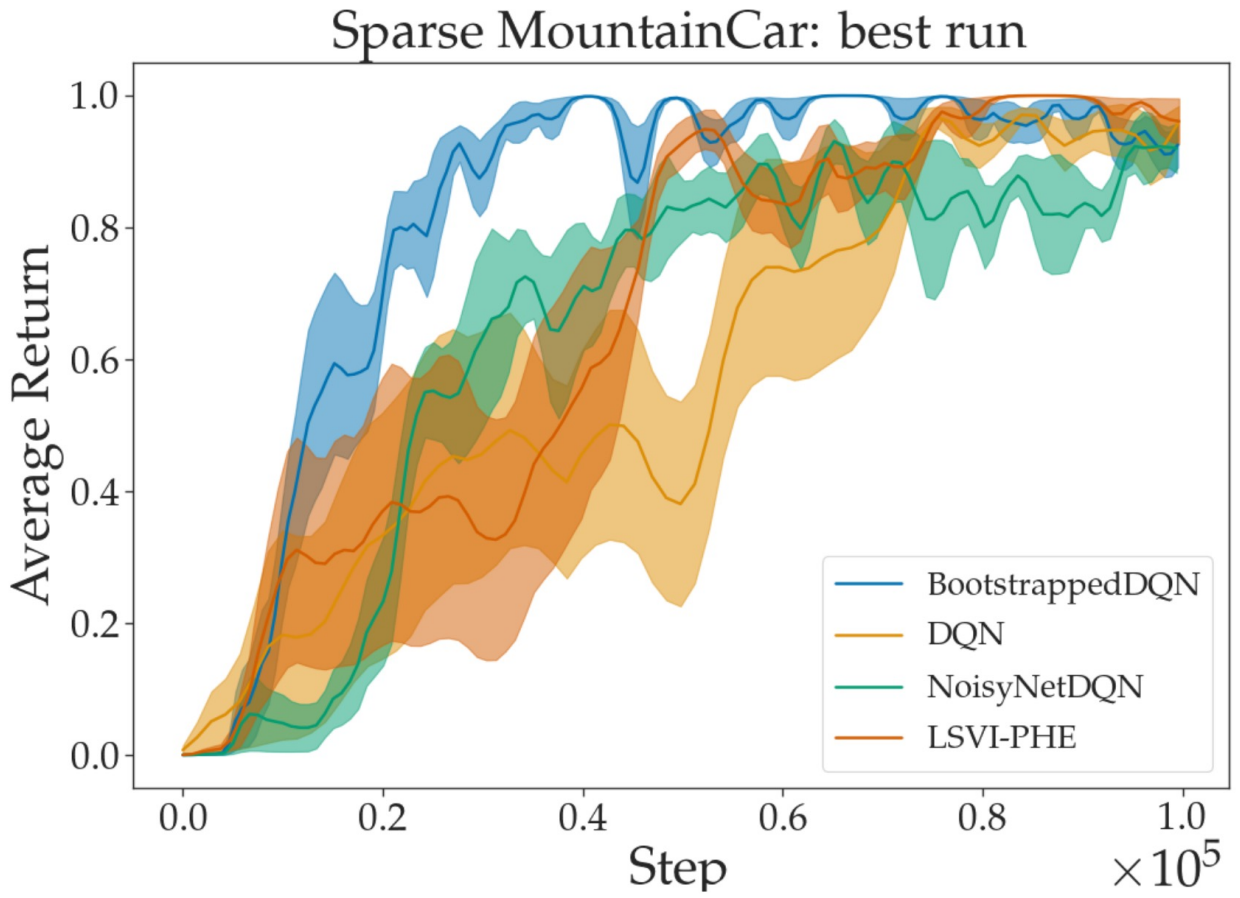
Deep Sea: M sensitivity





Mountain Car:





Summary

- Provably efficient RVF method for RL with general function approximation
  - Sublinear regret
  - Computationally efficient

- *Optimistic sampling allows us the unify OFU and Thompson Sampling*

Collaborators

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McGill, Mila

Lin Yang
UCLA

Paper Link:

[Ishfaq, Cui, Nguyen, Ayoub, Yang, Wang, Precup, Yang' ICML 2021] *Randomized Exploration for Reinforcement Learning with General Value Function Approximation*
https://arxiv.org/abs/2106.07841